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Baseline Uncertainty in Geometric Tolerance Inspection by Coordinate Measuring Machines: the Case of Position Tolerance with Maximum Material Condition[1]

Daniele ROMANO
University of Cagliari – Department of Mechanical Engineering
Piazza d'Armi – 09123 Cagliari - Italy
Phone: +39-070-6755710 Fax: +39-070-6755717
E-mail: romano@dimeca.unica.it

Grazia VICARIO
Politecnico of Turin – Department of Mathematics
Corso Duca degli Abruzzi 24 – 10129 Torino - Italy
Phone: +39-011-5647504 Fax: +39-011-5647599
E-mail: vicario@polito.it

Abstract. The paper concerns the analysis and the design of the measurement process by which position tolerances on mechanical parts are checked by Coordinate Measuring Machines (CMM). This measurement process is widely used in industry and conditions the good functioning of millions of components, assemblies and systems. CMMs inspect parts by exploring their surface at a small number of points and return the point Cartesian coordinates. Then data are numerically elaborated to estimate the position error. The analysis aims to evaluate measurement uncertainty as generated by two sources: the random error related to coordinate retrieval and the sampling error inherent to the way CMMs operate. By simulating random error via computer, the measurement process is fully recreated by a simulation model. Then an extensive computer experimentation, combining Montecarlo simulation and DOE, is performed. The study has revealed interesting statistical properties of the two-dimensional position error, which have useful practical implications. Another contribution of the paper is the use of the uncertainty analysis to design an efficient measurement process, namely one that attains a good trade-off between cost and accuracy. For a given total number of measurement points, their optimal allocation on the different part surfaces is provided.

Keywords: Measurement Quality, Uncertainty, CMM, Position Tolerance.

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1 INTRODUCTION

The relevance of industrial measurement processes to product quality is being increasingly recognized. This is witnessed by several facts. We will mention just two of them, one arising from technical standards, the other from research policy. The revised ISO standards 9000:2000 [1] brought about a substantial increase of directives related to management of the measuring process as compared with the 1994 issue. Within the framework of the V EC Research Program, the area of "Measuring and Testing" had by far the lion's share among the lines of research proposed, as e.g. the line "Competitive and sustainable development" raked up a full 50% within the 1999-2000 period.

A primary requirement for a Measurement Quality System is the assessment of the uncertainty related to any measurement result. Standards [2] consider two kinds of uncertainty: uncertainty of type A which has to be estimated using statistical procedures and uncertainty of type B whose estimate require other methods, such as accumulated technical knowledge, mathematical models and so on. Type A uncertainty is by far the most important one and this is why today's industrial metrology requires enormous support from statistics.

This paper deals with type A uncertainty in the evaluation of geometric conformance of mechanical components as measured with Coordinate Measuring Machines (CMM), widely used for such an application.

CMMs are increasingly being used both online for production control and for the inspection of finished products where tolerances apply, despite far from negligible costs; their unique capability and versatility of measurement more than justify investment. Both under operator control, for one at a time jobs, and under computer control for repetitive tasks, a CMM inspection is performed typically by sequentially logging in coordinates of points where a ball-end touch-fire probe contacts the surface of the piece under consideration. Point coordinates are referred to the machine as required. Exploration takes place generally along Cartesian coordinates, polar and cylindrical coordinates also being used whenever convenient. Point by point exploration is a sampling process, whose size is necessarily limited by time and cost; the inherent loss of information due to the sampling error calls for statistical evaluation, and may introduce substantial problems whenever several surfaces have to be explored on a given setup, as is the case for geometric tolerance control. Additionally, coordinates returned by the machine are affected by a random error whose effect on the final result of the control has to be assessed. These facts make the evaluation of measurement uncertainty on CMMs quite a challenging statistical problem, especially when complex controls are involved.

In the paper we address the analysis of uncertainty for position tolerance, one of the most complex cases of geometric tolerances. This tolerance is used whenever a part feature, e.g. a hole, a slot, a pocket, has to be precisely located with respect to other relevant part features, e.g. external faces or another hole, to make sure that the feature can mate with a corresponding feature on another part, like in the shaft-hole coupling. Therefore position tolerances play a key role when two or more components have to be assembled together. An example, which will serve as a reference throughout the paper, is provided in Figure 1.

Tolerance verification standards [3] state that conformity is declared, and a product accepted, whenever measurement result fall within the tolerance band deducted of uncertainty at both ends; on the other hand, non conformity is declared, and rejection occurs, whenever

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which the authors are members of, dedicated to furthering the use of statistics in industry and business.

measurement results fall outside of the tolerance band augmented by uncertainty at both ends. A procedure is specified, which must be approved explicitly by interested parties, leading to the definition of decision rules for results within the so-called "grey areas".

The most recognized method to estimate measurement uncertainty on CMMs is based upon substitution of the physical machine by a virtual machine. Given a set of results of an actual measurement cycle on the part at hand, a number of simulated cycles on the virtual machine are performed accordingly, and measurement uncertainty estimated on a statistical basis. Of course accuracy depends upon the model's approximation to the actual CMM performance. Model building needs the probabilistic characterization of coordinate measurement error within the operating space and the availability of correct algorithms for verifying the tolerance under consideration. Such an approach, first proposed by PTB, the German institute of metrology [4], and supported by EC within the IV Research Framework Program, was also used, albeit with some interesting differences, by other national metrology institutes, such as NIST in the US and IMGC in Italy.

However some significant research gaps still exist. Firstly, cases solved to date concern mainly dimensional tolerances and the simplest types of geometric tolerances, such as form tolerances (circularity, linearity, flatness), and orientation tolerances (parallelism, perpendicularity). As far as we know, no general solution has been given for the complex case of widely used position tolerances. We recall that they may be further complicated in the rather frequent instances in which allowable geometrical errors are interdependent, e.g. when the tolerance has to be verified when the so-called *Maximum Material Condition* (MMC) applies.

Furthermore, the design of the measurement process, namely the selection of number and spatial pattern of measurements, could benefit a lot from uncertainty analysis. In fact an efficient measurement cycle is one that provides a good compromise between uncertainty and the cost associated with it.

The methodology applied here is a combination of DOE and Montecarlo simulation applied to a computer model of the measurement process needed for position tolerance control. We demonstrate that this approach can gather precious information, which can be directly exploited to improve the process. Improvements are demonstrated in terms of uncertainty reduction as well as increased efficiency of the measurement cycle. Results presented here are the most recent of a study that has been going on for some years [5,6] and is now under evaluation for funding by the Italian Ministry for Scientific Research.

The paper is organized as follows: the next section describes the steps involved in the control of a typical position tolerance on CMM; in section 3 the simulation model of the measurement process is briefly outlined; then we present the methodology for the comprehensive uncertainty analysis; section 5 shows the main findings of the study and their implications on uncertainty reduction and process design; a discussion of both the methodology and results in the light of possible improvements in industrial practice on CMMs concludes the paper.

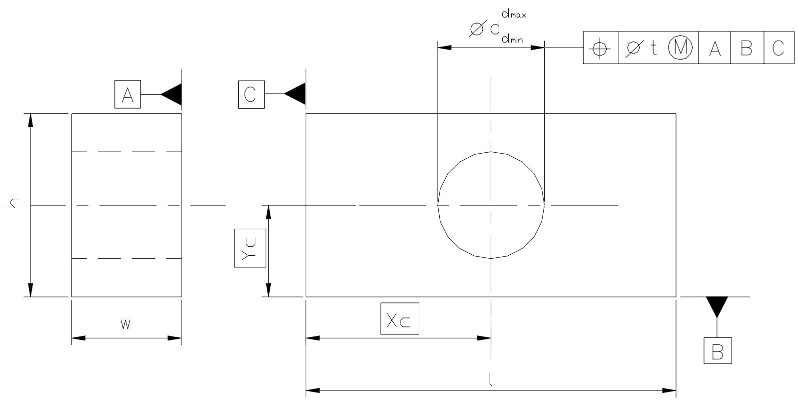

Figure 1. Drawing of a plate with position tolerance specification. All dimensions are denoted by letters to be used later in the parametric analysis.

2 POSITION TOLERANCE CONTROL ON CMM

An example of position tolerance is presented in the drawing in Figure 1, showing a plate with a circular hole in the middle. Dimensions and tolerances define nominal geometry as well as admissible errors on manufactured parts. All dimensions are denoted by letters which will be used in the parametric analysis later on. All necessary information about position tolerance is provided in the specification frame in the upper right corner. Checking part conformance to this tolerance requires to identify where the nominal axis and the actual axis of the hole are located; then a two-dimensional position error is calculated and contrasted with the tolerance specification, i.e. parameter t in the frame. The control procedure defined by the standards on geometric tolerances [7], as it is executed on a CMM, is summarized in the following steps:

1. Identification of a Cartesian Reference System. The system is established by three mutually orthogonal planes corresponding to the planes A, B, C referenced in the drawing. They are called datums and the system is the Datum Reference Frame (DRF). Datums are not the surfaces on the actual part, but perfectly flat mating counterparts of the actual surfaces. Having in mind the assembly function, datums correspond to imaginary planes which real faces are contacted with (envelope planes) following the referencing order. On CMMs planar datums are usually simulated by high-quality flat faces of an *ad-hoc* fixing device. A set of points is probed on the horizontal face of the device and datum A is estimated by the Orthogonal Least Squares (OLS) method. Then datum B is estimated analogously with the additional constraint of being perpendicular to datum A; datum C is finally estimated with a double othogonality constraint (to both A and B). Notice that the order of datums is not irrelevant. The intersection of A, B and C defines the origin of the DRF.
2. Identification of the nominal hole axis. It is a line perpendicular to datum A, parallel to, and offset by Yc and Xc from B and C respectively.
3. Identification of the actual hole axis. It is defined as the axis of the largest size cylinder (perpendicular to datum A) able to enter the hole; it must be determined by calculation after the inspection of a sample of points on the hole surface. The simplest way to do it is to project on datum A all the inspected points and then determine the center of the largest circle inscribing the projected points.
4. Calculation of the position error. As both axes previously identified are perpendicular to datum A, we can define position error as a two-dimensional vector, $\mathbf{e_p}$, going from the

nominal hole axis to the actual hole axis onto the datum A, see Figure 2. Final part acceptance is decided if

$$| \mathbf{e_p} | \leq t/2$$

Needless to say, this is the deterministic acceptance rule that must be replaced by a probabilistic one which takes into account the uncertainty contained in the position error. The decision rule is further complicated whenever the MMC modifier is present. It is denoted by a circled M in the tolerance frame. It introduces a relation between the position error on the hole position and the dimensional error on the hole diameter, by allowing for a larger position error whenever the actual diameter is larger than the minimum possible (d_{min}), which actually identifies the condition where the part's material is maximized. In such a case an equivalent position error is defined and the decision rule modified as:

$$| \mathbf{e_{peq}} | = | \mathbf{e_p} | - (d_{act} - d_{min}) \leq t/2$$

where d_{act} is the actual hole diameter as measured by the CMM. The new position error $\mathbf{e_{peq}}$ is smaller than $\mathbf{e_p}$ by the quantity ($d_{act} - d_{min}$), called tolerance bonus. This is helpful in reducing the number of scrapped parts but increases the uncertainty of the control result by the contribution of the additional term ($d_{act} - d_{min}$). Probabilistic acceptance rules will be introduced in section 5.

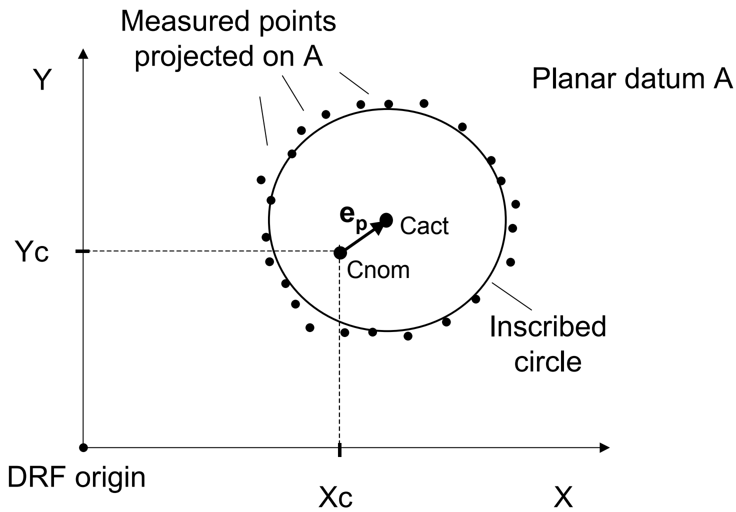

Figure 2. Two-dimensional position error in the plane of datum A.

3 THE SIMULATION MODEL OF THE MEASUREMENT PROCESS

The objective of the paper is to analyze in depth uncertainty of position error, investigating how it is affected by the measurement process and the part's geometry. Such a comprehensive analysis can only be done on a simulation model of the whole measurement process. The simulation model that we adopted has the following characteristics:

1. The part has no error. Its geometry is exactly as represented in Figure 1. Hence uncertainty, studied in such a special case, is a baseline uncertainty, as only the measurement random error is active. However this is not a very heavy restriction. As far as identification of datums is concerned, measurements are not taken on the part's faces but on the faces of the fixing device, which are very close to ideal planes. As for the hole,

points are probed on the real part surface. In principle, uncertainty should be studied case by case. Nevertheless, systematic departure from the cylindrical form may well be identified and accounted for, so that again, the case of a known geometry plus random error occurs. Accidental form errors are not handled by the analysis. But this is a more general problem concerning CMMs that, working with small samples, is always subject to loose important irregularities in the part shape, especially if they are located in a small area.

2. Measurement errors on the coordinates returned by the CMM is considered additive and is described by i.i.d. normal random variables with zero mean and common variance, $\sigma^2 = 0.005^2$. Systematic errors due to the machine (form errors in the slides, error linked to the approach angle, etc.) are not considered here as they can be identified and corrected, see for example [8].
3. The datum planes and the circle in Figure 2 are computed by OLS estimation. This is because the part has no error and applying the envelope principle would cause bias, as shown in [5].

For more details on the estimation algorithms refer to [5] for the datum planes and to [6] for the circle. They are available as a Matlab code.

4 UNCERTAINTY ANALYSIS

In this section we present the methodology for uncertainty analysis and the main results. We apply a combination of Montecarlo simulation and designed experimentation on the simulation model of the measurement process. Basically Montecarlo simulation is needed to extract reliable sample statistics for process output; whilst designed experimentation is able to try the process under a wide set of conditions in an efficient way. Technically, for every trial of the planned experiment a Montecarlo simulation is run with 10^4 repetitions. During repetitions the simulation model is not changed apart from the random measurement error. The experimental factors are of two kinds, control factors and blocking factors. The former belong to the measurement process, the latter to the part geometry. The four control factors are the number of measurement points on the surfaces related to datums A, B and C and on the hole surface. The four blocking factors are the thickness of the plate, w, the dimensions Xc and Yc, defining how much the nominal hole center is shifted out from the DRF origin and the hole diameter, d. The experiment is a 273 run Central Composite Design, with three levels per factor. It allows for fitting a quadratic predictive model that has been used for design optimization in the next section. Table 1 resumes factors and their levels.

Table 1. Factors and levels of the experiment, with their coded values.

Labels	**Control factors**	**Levels (codes)**		
n_A, n_B, n_C, n_H	Number of points measured on surfaces A, B, C and on the hole	4 (-1)	9 (0)	16 (1)
	Blocking factors			
w	Plate thickness	25 (-1)	50 (0)	75 (1)
Xc	Horizontal boxed dimension	50 (-1)	100 (0)	150 (1)
Yc	Vertical boxed dimension	50 (-1)	100 (0)	150 (1)
d	Hole diameter	25 (-1)	50 (0)	75 (1)

Nominal measurement points on the planes and the hole are positioned in a regular pattern and their number is in the range of 4 to 16 per surface. On the planes, points form Cartesian arrays (2x2, 3x3, 4x4) with a clearance along the surface edges. On a generic plane z=cost, points are identified by pairs (x_i, y_j):

$$x_i = L_x (0.05 + 0.9\,(i-1)/(n^{1/2}-1)), \qquad i = 1 \text{ to } n^{1/2}$$
$$y_j = L_y (0.05 + 0.9\,(j-1)/(n^{1/2}-1)), \qquad j = 1 \text{ to } n^{1/2}$$

where L_x and L_y are the dimensions of the surface and n is the total number of measurement points. On the hole, surface points are identified by pairs (ϕ_k, $z_{k'}$) defining a random helix in a cylindrical system:

$$\phi_k = 2\pi\ (k-1)/n$$
$$z_k = w\,(0.05 + 0.9\,(k-1)/(n-1)), \qquad k = 1 \text{ to } n$$

where k' is a random permutation of k. Selection of these patterns seems quite reasonable as they accomplish a nearly uniform coverage of the surface. Anyway the effect of different patterns on the uncertainty, which would also be valuable information, is not considered in this study.
The outcome of the simulation model is the position error $\mathbf{e_p}$, or $\mathbf{e_{peq}}$ when the MMC holds. Although the decision rule involves only the modulus of the position error, it is informative to study its phase too, as it will be clearer further on. A convenient representation for position error is the polar one, $\mathbf{e_p} = \rho e^{i\theta}$ and a suitable measure of uncertainty for $\mathbf{e_p}$ is the area of the confidence region of the two-dimensional random variable (ρ, θ) at a (1-α) significance level. It is possible to find a boundary $\rho(\theta)$, with $0 \le \theta \le 2\pi$, of a conjoint confidence region $I_{1-\alpha}$ by using conditional distribution $f_{\rho|\theta}$ and marginal f_θ:

$$\int_I f_{\rho,\vartheta}(r,t)\,drdt = \int_0^{2\pi}\int_0^{\rho(\vartheta)} f_{\rho|\vartheta}(r,t)\,f_\vartheta(t)\,drdt = 1-\alpha$$

A numerical solution is provided by using the empirical distributions f_θ and $f_{\rho|\theta}$, which can easily be derived from Montecarlo simulations. Let $A_{(1-\alpha)\ \times 100}$ denote the area of the confidence region. As an example, in Figure 3 the empirical 95% confidence region is shown for two experimental points, the ones with all factors at the low level (dashed boundary) and the ones with all factors at the high level (continuous boundary).

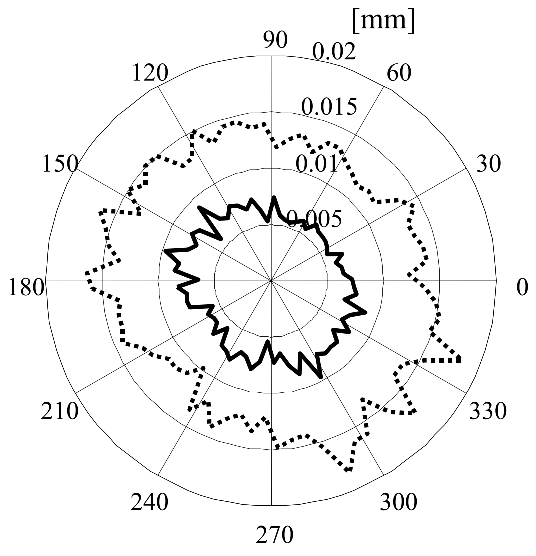

Figure 3. Empirical 95% confidence region of $\mathbf{e}_{peq}$ for two experimental settings. All factors have the low level (-1) for the region with dashed boundary, the high level (+1) with continuous boundary. A_{95} is 599 μm^2 and 153 μm^2 respectively.

Interestingly variability in $f_{\rho|\theta}$ is dependent on direction. This is valuable information that we can exploit when assessing uncertainty. The phase of the position error results from the measurement process at no additional cost. It is no use when applying the deterministic decision rule where only the modulus appears. However it is very useful when uncertainty is taken into account in the acceptance decision. The best probabilistic acceptance rule with a 95% confidence level is:

$$| \mathbf{e_p} | \leq t/2 - P_{95}(\rho|\theta)$$

where $P_{95}(\rho|\theta)$ is the 95th percentile of the random variable $\rho|\theta$, being θ the phase resulting from the measurement process. In the case of MMC the rule becomes:

$$| \mathbf{e}_{peq} | = | \mathbf{e_p} | - (d_{act} - d_{min}) \leq t/2 - P_{95}(\rho-(d_{act} - d_{min})|\theta)$$

It is not convenient to use a rule which is independent of θ, as small gain in simplicity is more than counteracted by decisions which are more prone to error.
Analysis of all effects from the 2^8 full factorial reveals that the first five main effects are predominant, with only two double interactions stemming out ($n_A w$ and $n_B n_C$). This is clearly shown in Figure 4 by the normal plot of all factorial effects on A_{95} for $\mathbf{e_p}$. The same pattern of effects applies with $\mathbf{e}_{peq}$; the only difference is an increased main effect of n_H which, in both cases, is the largest effect, see Figure 5. This is in line with the fact that additional uncertainty in $\mathbf{e}_{peq}$ is due to the measure of the actual hole's diameter d_{act}, which of course is dependent on n_H only. The increase in A_{95} with MMC is not negligible, ranging within the experiment from 6.5% (n_H=16, n_A=4, n_B=4, n_C=4, w=25, Xc=150, Yc=50, d=25) to 33.6% (n_H=4, n_A=16, n_B=16, n_C=16, w=25, Xc=150, Yc=50, d=25) with an average figure of 18.3%.
An outstanding result is that the numbers of measurement points on the three datum surfaces affect uncertainty almost at the same extent. This finding disproves the belief, very popular among practitioners, that measurement points must be allocated on datum surfaces A, B and C according to a ratio 3:2:1. This practice is valid to reduce the uncertainty in the determination of the DRF origin, as was shown in [5], but it is not in the position tolerance control.

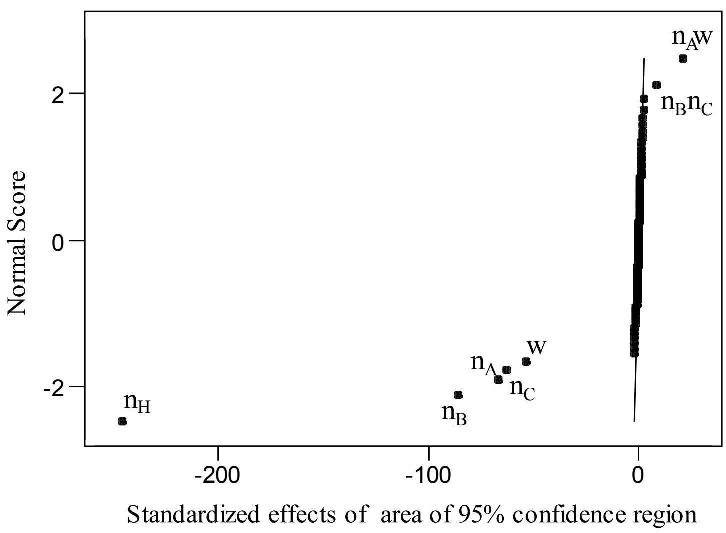

Figure 4. Normal plot of factorial effects on the area of the 95% confidence region of $\mathbf{e}_{peq}$

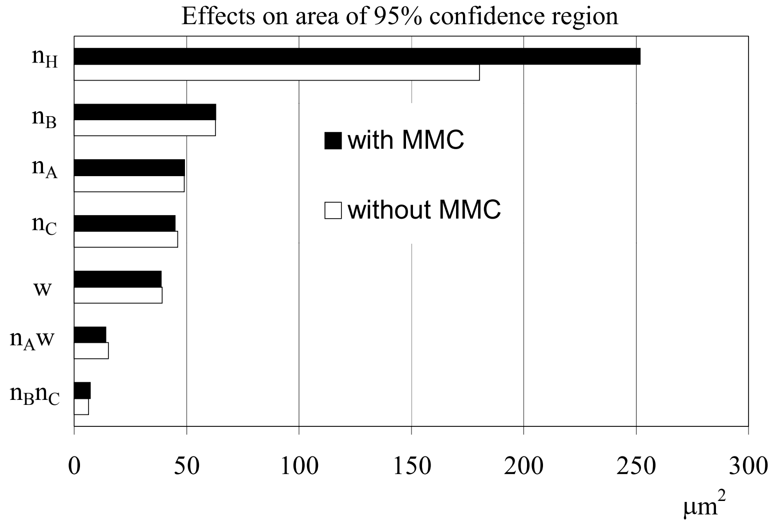

Figure 5. Pareto diagram of the largest effects on the area of the 95% confidence region of $\mathbf{e_{peq}}$ (white bars) and $\mathbf{e_{peq}}$ (black bars).

Notably only the plate thickness w is active among the geometrical factors. However, a closer scrutiny shows that dimensions Xc and Yc, though not influencing the area of the confidence region, do influence its shape. In fact, a cigar-like shape becomes more apparent when Xc and Yc decrease, i.e. when the hole center is closer to the DRF origin. In Figure 6 the confidence region with all factor at the low level (inner region) can be compared with the confidence region with the same factor levels, except Xc and Yc that are both zero. Although the latter case is not realistic (the hole center is in the origin) it demonstrates a very clear change in shape.

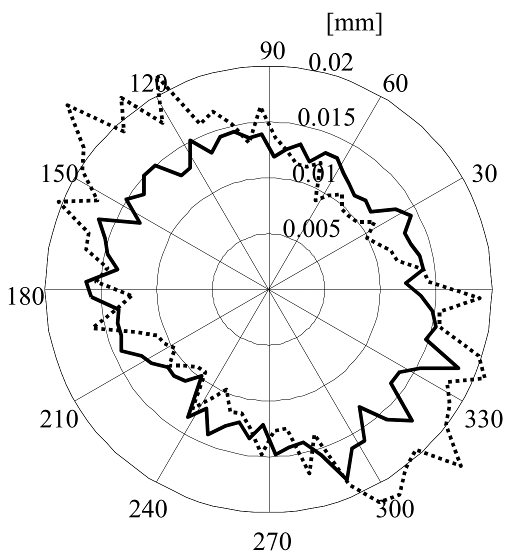

Figure 6. Empirical 95% confidence regions of $\mathbf{e_{peq}}$ showing a prominent cigar-like shape when the hole center is located on the DRF origin. Xc and Yc are 50 mm for the region with continuous boundary, 0 mm for the region with dashed boundary. All other factor are at the low level.

5 DESIGNING AN EFFICIENT MEASUREMENT PROCESS

Here we describe how the previous analysis can be used to design an efficient measurement cycle on the machine. From the outcome of the surface response design a quadratic predictive model can be estimated for our proxy of uncertainty as a function of both control and blocking factors:

$$\hat{A}_{95} = f_Q(n_H, n_A, n_B, n_C; w, Xc, Yc, d)$$

Now we define a simple optimization problem aimed at finding, for a given part geometry, the setting for the control factors that minimizes the uncertainty function $\hat{A}_{95}$ under the constraint of a given total number of measurement points, n_{TOT}. Denoting by **x** the unknown vector of the control variables and by $\mathbf{b_0}$ the fixed vector of blocking parameters relative to the given part, the optimization problem can be stated as:

$$\begin{aligned} &\min_{\mathbf{x}} \hat{A}_{95}(\mathbf{x}; \mathbf{b_0}) \\ &\text{subject to:} \\ &(1111)\mathbf{x} = n_{TOT} \\ &\mathbf{LB} \le \mathbf{x} \le \mathbf{UB} \\ &\mathbf{x} \text{ integer} \end{aligned}$$

where **LB** and **UB** are vectors of lower and upper bounds for control variables. The problem entails the minimization of a non linear function under a linear constraint and bilateral bounds for the integer-valued solution.

We present two examples of process optimization; in the first, the solution is also compared with what would be a typical setting in an industrial lab. In both cases part geometry is defined by $\mathbf{b_0}$=(75mm 100mm 100mm 50mm)^T. $\hat{A}_{95}$ is identified by linear regression on MINITAB in the case of MMC. Its fitting capability is quite good ($R^2_{adj} = 99.8\%$). In the case at hand **LB**=(4 4 4 4)^T and **UB**=(16 16 16 16)^T as it is safer to search for the solution within the experimental range of control variables where $\hat{A}_{95}$ is estimated. The two cases have n_{TOT} equal to 32 and 21 respectively. Table 2 reports the main results.

Table 2. Solution of the optimization problem defining an efficient measurement process. In the first case results are contrasted with those pertaining to a typical industrial setting.

			Solution					
Case	n_{TOT}	Solution type	n_H	n_A	n_B	n_C	$\hat{A}_{95}$ [μm^2]	$\bar{\rho}_{95}$ [μm]
#1	32	Optimized	14	4	8	6	217	8.3
		Typical	8	12	8	4	355	10.6
#2	21	Optimized	9	4	4	4	336	10.3

The most noticeable discrepancy is that the typical lab solution allocates more measurement points on the first datum surface whereas the optimized solution allocates more measurement points on the hole surface. Moreover, the importance of measurement points on A, B and C is more balanced compared with the 3:2:1 rule of thumb. In the last column of the table, an average measure of uncertainty of the modulus of position error is obtained by calculating the radius of an equivalent circular confidence region having an area equal to A_{95}:

$$\overline{\rho}_{95} = \sqrt{\frac{A_{95}}{\pi}}$$

We observe that the optimized solution exhibits a reduction of 39% in terms of A_{95} and 22% in terms of $\overline{\rho}_{95}$. In case #2, even if 11 measurement points less than in case #1 are used, the optimized uncertainty is still inferior to the one obtained from the typical solution in case #1.

6 FINAL REMARKS

The paper gives an insight on how statistical methods, and experimental design in particular, can prove very valuable to analyze measurement uncertainty and to design an improved measurement process. The research in this field is expected to become more and more productive as a strong link is recognized between a sound metrological approach and product quality.

Here we have focused on uncertainty in the control of position tolerances on mechanical parts operated by a CMM. A baseline uncertainty is calculated working on a simulation model of the measurement process. A brief discussion on the main findings of the study and future prospects is given below pointwise.

1. The size of the baseline uncertainty in the observed sample space is not negligible. The largest 95-th percentile of the modulus of position error conditioned on the phase can be as much as three times the standard deviation of the random measurement error (up to four times when MMC is present). Incorrect decision about acceptance/rejection may well be incurred if commonly used deterministic rules are applied.
2. Uncertainty is peculiarly dependent upon the direction of position error. The central symmetry of the tolerance zone, as dictated by the standards, is lost. The knowledge of the phase of position error allows for a more accurate assessment of uncertainty. If only the modulus is considered, as it is in industrial practice, uncertainty is either underestimated or overestimated depending on the phase of position error. Therefore, a consistent evaluation requires a far more involved software than that installed on most CMMs.
3. A comprehensive analysis assessing how all factors involved in the measurement process affect uncertainty is crucial to design efficient measurement cycles. The combination of experimental design and Montecarlo simulation proves effective to provide such an analysis. It allows to expose some false beliefs of engineers, largely misused in practice. Moreover, based on the analysis, an optimized process can be designed case-by-case. A customized selection of process parameters is possible depending on the available budget and the part geometry at hand. The paper describes a method to optimally allocate the measurement points on the different part surfaces. However, the design of the optimal location of measurement points is not dealt with here and it is one of the subjects we plan to study further.

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